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A New Approach to Check the Validity of Arguments without Using Rules of Inference

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***Abstract*—Check the validity of an argument is a very important topic in the study of Mathematical logic of discrete mathematics. It has plenty of applications in mathematical and computer sciences. It is observed that in the predicate logic, it is difficult for students to validate the arguments using the rules of inference, as it is difficult to remember all the rules. In this article, a new method is introduced to check the validity of arguments in the quantified statements. In this method, students need not to remember the rule of inference. So it is easy to validate the argument using the method discussed in this paper. Two examples are solved using the new methods as well as the old method for comparison. Old method is by using inference rules in which students need to remember all the rules. While, in the new approach of solving need not any rules of inference.**

***Index Terms*—Logic; rules of inference; quantification; modus ponens; valid arguments; predicate**

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I. INTRODUCTION

The importance of discrete mathematics in both research and in education has been highly marked and extensively studied in the literature. It has an important basic position in the theoretical system of computer science. By learning the knowledge of discrete mathematics, students can cultivate their subject quality and master the correct subject methods. Discrete mathematics is an interesting course to teach and to learn at the freshman and sophomore level for several reasons. Its content is Mathematics, but most of its applications and more than half its students are from computer science. Thus careful motivation of topics and previews of applications are important and necessary strategies. In addition, the student is often expected to develop an important new skill: the ability to write mathematical proof. This skill is excellent training for writing good computer programs.

Intuitively, logic is the discipline that considers the methods of reasoning. It provides the rules and techniques for determining whether an argument is valid or not. In everyday life, we use reasoning to prove different points. For example, to prove to our parent that we passed an exam, we might show the score card. Similarly in mathematics and computer science, mathematical logic is used to prove results. To be specify, in mathematics we use logic or logical reasoning to prove theorems, and in computer science we use logic or logical reasoning to prove the correctness of programs and also to prove theorems.

Predicate logic builds heavily upon the ideas of proposition logic to provide a more powerful system for expression and reasoning. An important use of predicate logic is found in computer databases and the more general notion of "knowledge base", defined to be a database plus various computation rules. Predicate logic has been used to increase precision in describing and studying structures from linguistics and philosophy to mathematics and computer science. In the computer science field, predicate logic has a wide variety of applications and hence is very important. It is used in system specifications, circuit designing, logical puzzles, etc. Apart from this, it can also be used in translating English sentences to mathematical statements and vice-versa.

David (1985) has shown that how to use discrete mathematics in encouraging an investigatory approach in mathematics teaching in schools with few interesting examples. David et al. (1995) had recommended teaching introductory discrete mathematics by first teaching propositional and predicate logic and then using it as tool throughout the study of later topics of discrete mathematics. Selden et al. (1995) have checked the ability of the undergraduate students to unpack the informally written mathematical statements into the language of predicate calculus. Janet et al. (2016) had described teaching an introductory discrete mathematics course entirely from student projects based on primary historical sources. case studies of four projects that cover the content of a one-semester course, and mention various other courses that we have taught with primary source projects is presented in their paper. The interesting analysis for the teaching of discrete mathematics as a backbone of computer science is given by Mamta (2019).

Xue et al. (2019) have discussed the teaching concept of hybrid learning and collaborative learning through real-time feedback teaching interaction tools, which realizes teaching feedback and interaction with the help of the auxiliary platform. Their research combines mobile learning with traditional courses to realize the teaching effect of learning at any time. The origin of first-order logic, the importance of quantified statements, the interplay between syntax and semantics, truth in an interpretation and logical validity are discussed in detailed by Durand-Guerrier (2022). In a survey paper by Sandefur et al. (2022), several convincing arguments that discrete mathematics should be more strongly considered in mainstream mathematics education are given. These relate to its accessibility, the acquisition of general competencies, affective benefits, and the utility of knowledge of discrete topics in the modern world.

At school level, educational opportunities of teaching discrete mathematics and mathematical modeling are assessed by Gilbert et al. (2022). They also discussed the possibilities and opportunities for using examples from the field of discrete mathematics to acquire modeling

competences and to foster the linkage of mathematical modeling and discrete mathematics at school level. Colipan et al. (2022) have discussed how problems issued from discrete mathematics are useful in developing attitudes, knowledge and skills required for scientific mathematical activity in the classroom. Vera-Zambrano et al. (2022) have analyzed that the development of mathematical logic skill is very important for students to understand the other mathematical concepts. Ramanujam (2023) proposed a set of Big Ideas of mathematical logic relevant to education: for instance, the notion of truth relative to a structure, construction of models for a set of sentences, consistency of procedures used for computation, comparison of algorithms, and so on. He also discussed how they can significantly enrich mathematics and computing science curriculum and pedagogy, clearing up students' common misconceptions, referring to some experiences with high school mathematics teachers and students.

To check the validity of argument is very important topic in the study of Mathematical logic of discrete mathematics. It has plenty of applications in mathematical and computer sciences. It is observed that in the predicate logic, it is difficult for students to validate the arguments using the rules of inference (ROI), as it is difficult to remember all the rules. In this article, a new method is introduced to check the validity of arguments in the quantified statements. In this method, students need not to remember ROI. So it is easy to validate the argument using the method discussed in this paper. Two examples are solved using the new methods.

II. FUNDAMENTALS OF MATHEMATICAL LOGIC

Before going to the new technique to prove the validity of an argument, it is important to know some basic terminology of mathematical logic. Below are few definitions and explanations of some terms of propositions and predicates (Kolman et al. (2007) and Malik et al. (2009)).

Definition 2.1.

Propositional logic is a mathematical system for reasoning about propositions and how they relate to one another. Every statement in propositional logic consists of propositional variables combined via logical connectives. A statement or a proposition is a declarative sentence that either true or false, but not both.

Remark 2.1.

Propositional logic enables us to the following.

(i) Formally encode how the truth of various propositions influences the truth of other propositions.
(ii) Determine if certain combinations of propositions are always, sometimes, or never true.
(iii) Determine whether certain combinations of propositions logically entail other combinations.

Remark 2.2.

Limitations of Propositional logic are given below.

(i) Statements that hold for many objects must be enumerated.

(ii) Statements that define the property of the group of objects.

Remark 2.3.
Predicate logic Remedies the limitations of the propositional logic.
(i) Explicitly models objects and their properties.
(ii) Allows making statements with variables and quantifying them.

Definition 2.2.
Predicate Logic deals with predicates, which are propositions containing variables. A predicate is an expression of one or more variables defined on some specific domain. A predicate with variables can be made a proposition by either assigning a value to the variable or by quantifying the variable.
There are two types of quantifier; Universal quantifier (all, any, each, every) and Existential quantifier (there exists, some, at least one).

Definition 2.3.
Let P(x) be the predicate and let D be the domain of the disclosure. The universal quantification of P(x) is a statement can be written as below.
For all x, P(x).
In notation, the universal quantification of P(x) can be written as below.
$\forall x \ P(x)$.
The symbol used to denote the adjectives for all (for every) is $\forall$, and it is called the universal quantifier.

Definition 2.4.
Let P(x) be the predicate and let D be the domain of the disclosure. The existential quantification of P(x) is a statement can be written as below.
There exists x, P(x).
In notation, the existential quantification of P(x) can be written as below.
$\exists x \ P(x)$.
The symbol used to denote "there exists" is $\exists$, and it is called the existential quantifier.

Definition 2.5.
An argument is a sequence of statements. All statements in an argument except for the final one are called premises (or assumptions, or hypothesis). The final statement is called the conclusion. The symbol "$\therefore$" which is read "therefore" is normally placed just before the conclusion.

Definition 2.6.
An argument form is valid if whenever true statements are substituted in for the statement variables the conclusions is always true. To say an argument is invalid means that it is not valid.

The main point regarding a valid argument is that it follows from the logical form itself and has nothing to do with the content. When a conclusion is reached using a valid argument, we say the conclusion is inferred or deduced from the premises.

Remark 2.4.
Some valid argument forms (rules of inference) are given below.
(i) The following argument form is called Modus Ponens.
$p \to q$,
p,
$\therefore q$.

The statement formula of the above form is $\big((p \to q) \wedge p\big) \to q$. This is a tautology, so this is a valid argument form.
(ii) The following argument form is called Modus Tollens.
$p \to q$,
$\neg q$,
$\therefore \neg p$.

The statement formula of the above form is $\big((p \to q) \wedge \neg q\big) \to \neg p$. This is a tautology, so this is a valid argument form.
(iii) The following argument form is called Dilemma.
$p \vee q$,
$p \to r$,
$q \to r$,
$\therefore r$.
It is also a valid argument form.
There are many other forms in the list of some valid argument forms such as, disjunctive syllogism, hypothetical syllogism, absorption, conjunctive simplification, additions (disjunctive addition), conjunction (conjunctive addition). Few of them are mentioned in Table 1.

III. NEW METHOD TO CHECK THE VALIDITY OF ARGUMENTS

Author had developed the new method to check the validity of argument for predicate logic problems. To apply this method, students need not to remember ROI. Only some rules of equivalence are required to apply to determine whether or not a given argument is valid. Therefore, the new method can be easier to apply for the school level and university level students. Also, it may be pedagogically convenient that the student does not have to deal with all the ROI.
Here, in the present paper, two well-known arguments (examples) are considered and the validity of both the argument is checked using the new method (it is mentioned as Method 2). The same

argument problems are solved using the ROI in reference books (it is mentioned as Method 1). Below are the examples and the solution by method 1 and by method 2 (new method).

Example 3.1.
All men are mortal.
Socrates is a man.
Therefore, Socrates is a mortal.
The above statements can be formalized as follows.
$P(x)$ is "x is a man".
$Q(x)$ *is* "x is mortal".
So we can write the above argument in the following form.
All men are mortal : $\forall x \quad P(x) \to Q(x)$,
Socrates is a man : P(s),
Therefore Socrates is a mortal : $\therefore Q(s)$.
The domain of disclosure is the set of all real numbers.
Method 1. To verify the validity of the above argument, we now consider the following sequence of formulas (Malik et al. (2009)).

$B_1 : \forall x \quad P(x) \to Q(x)$; hypothesis,
$B_2 : P(s) \to Q(s)$; by the rule of inference US,
$B_3 : P(s)$; hypothesis,
$B_4 : Q(s)$; B_2, B_3, B_4 logical argument by Modus Ponen.

Hence the given argument is avalid argument.

Method 2. (New method) Easy method to prove the above argument is valid.

$$
\begin{aligned}
&(\forall x (P(x) \to Q(x))) \wedge P(s) \to Q(s) \\
&\equiv (P(s) \to Q(s)) \wedge P(s) \to Q(s), \\
&\equiv (\neg P(s) \vee Q(s)) \wedge P(s) \to Q(s), \\
&\equiv [\neg P(s) \wedge P(s)] \vee [Q(s) \wedge P(s)] \to Q(s), \\
&\equiv F \vee [Q(s) \wedge P(s)] \to Q(s), \\
&\equiv [Q(s) \wedge P(s)] \to Q(s), \\
&\equiv \neg [Q(s) \wedge P(s)] \vee Q(s), \\
&\equiv \neg Q(s) \vee \neg P(s) \vee Q(s), \\
&\equiv T \vee \neg P(s), \\
&\equiv T.
\end{aligned}
$$

Hence the given argument is a valid argument.

Example 3.2.

All dogs fetch,
Kitty does not fetch,
Therefore, Kitty is not a dog.
The above statements can be formalized as follows.
$P(x)$ is "x is a dog".

$Q(x)$ is "x fetch".

So we can write the above argument in the following form.

All dogs fetch : $\forall x \quad P(x) \to Q(x)$,

Kitty does not fetch : $\neg Q(k)$,

Therefore, Kitty is not a dog. : $\therefore \neg P(k)$.

The domain of disclosure is the set of all real numbers.

Method 1. To verify the validity, we now consider the following sequence of formulas (Malik et al. (2009)).

$B_1 : \forall x \quad P(x) \to Q(x)$; hypothesis,

$B_2 : P(k) \to Q(k)$; US,

$B_3 : \neg Q(k)$; hypothesis,

$B_4 : \neg P(k)$; B_2, B_3, B_4 logically valid argument by Modus Tollen.

Method 2. Easy method to prove the above argument is valid

$$(\forall x (P(x) \to Q(x))) \wedge \neg Q(k) \to \neg P(k),$$
$$\equiv (P(k) \to Q(k)) \wedge \neg Q(k) \to \neg P(k),$$
$$\equiv (\neg P(k) \vee Q(k)) \wedge \neg Q(k) \to \neg P(k),$$
$$\equiv [\neg P(k) \wedge \neg Q(k)] \vee [Q(k) \wedge \neg Q(k)] \to \neg P(k),$$
$$\equiv [\neg P(k) \wedge \neg Q(k)] \vee F \to \neg P(k),$$
$$\equiv [\neg P(k) \wedge \neg Q(k)] \to \neg P(k),$$
$$\equiv \neg [P(k) \vee Q(k)] \to \neg P(k),$$
$$\equiv [P(k) \vee Q(k)] \vee \neg P(k),$$
$$\equiv [P(k) \vee \neg P(k)] \vee Q(k),$$
$$\equiv T \vee Q(k),$$
$$\equiv T.$$

IV. COMPARISON

To check the validity of argument with respect to predicate logic is sometimes a quite difficult task for a researcher. As to check the validity one has to remember all ROI. Therefore, author has developed a new method to check the validity of argument without using ROI.

The solution given in the literature (Malik et al. (2009)) is mentioned as a method 1. It is also correct and interesting. These solutions include fewer steps but, it used the ROI. If researcher is not being able to recall the proper form of ROI, then it is difficult to solve. The new method which is mentioned as a method 2 is comparatively lengthy but, it is free from any ROI. So it is easy to apply. Author has no conflict of interest with the method 1.

V. CONCLUSION

Discrete mathematics plays an important role in supporting the basic theory in the curriculum system of computer science and technology, and plays an important role in improving the quality

and ability of students. It is observed that, while teaching the logical arguments and its validity to the students, it is easy for them to prove whether the argument is valid or not using the truth table. But to prove the same example of an argument using the rules of inference, student face difficulty in recalling the rules.

This paper introduces a new technique to check the validity of arguments as the teaching content of predicate logic of discrete mathematics course. An argument contains the quantified statement can check for validity using the rules of inference. But there are many rules of inference with formula and particular names. While proving the argument for validity, it is difficult task for students to recall this rules therefore they cannot apply these rules to universal or existential quantifiers. To overcome from this difficulty, student can apply the new method discussed in the present paper. It will be beneficial to apply on quantified statements. This new method allows us to prove more complex arguments that contain a specific number of variables. It will be useful for the school-level as well as university-level students.

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Table1. Few Rules of inference (https://images.app.goo.gl/zF4V5PCXc82LC9wY8)

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \wedge (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	Resolution